Pearson Edexcel

Mark Scheme (Results)

Summer 2024

Pearson Edexcel International Advanced Level
In Pure Mathematics P3 (WMA13) Paper 01

Question Number	Scheme	Marks
1 (a)	$(5,10)$	B1 B1
		(2)
(b)	Attempts to solve e.g. $-2(x-5)+10...6x \Rightarrow x..."\frac{5}{2}"$	M1
	$x<\frac{5}{2}$ o.e.	A1
		(2)
(c)	$(7,30)$	B1ft, B1ft
		(2)
		(6 marks)

Condone missing brackets around pairs of coordinates provided the meaning is clear

(a) If there is a contradiction between coordinates on the graph, by the question and in the main body of the work then the work in the main body of the work takes precedence.

B1: One correct coordinate. Allow as $x=5$ or $y=10$ Do not allow them to be the wrong way round

B1: Both coordinates correct. Allow as $x=5, y=10$

(b) Answers with no working scores 0 marks

M1: Attempts to solve a correct equation or inequality with at least one intermediate stage of working. Allow this mark even if there are additional equations/inequalities. The modulus signs must have been removed.

Alternatively, they may attempt to square both sides and then attempt to solve the quadratic (usual rules apply). If via a calculator the root(s) must be correct (you may need to check these)

Condone slips including when squaring and multiplying out

$|x-5|...3x-5 \Rightarrow x^2-10x+25...9x^2-30x+25 \Rightarrow 0...8x^2-20x \Rightarrow x...$

In either method condone slips in their rearrangement and do not be concerned by the direction of the inequality sign for this mark.

A1: $x<\frac{5}{2}$ o.e. **only**. isw if they attempt to simplify their fraction incorrectly. Must be identified as their only inequality e.g. circling, underlining.

Allow other forms such as $x \in \left(-\infty, \frac{5}{2}\right)$ Do not accept $x \le \frac{5}{2}$

Do not withhold this mark for incorrect use of inequality signs in earlier work provided the final answer is correct.

(c)

B1ft: One correct coordinate. Allow as $x=7$ or $y=30$

Follow through on their $((a)+2,(a)\times 3)$ Allow as $x=(a)+2$ or $y=(a)\times 3$ (**Does not have to be evaluated**)

B1ft: Correct coordinates $(7,30)$ Allow as $x=7, y=30$

or correct ft $((a)+2,(a)\times 3)$. Allow as $x=(a)+2,\ y=(a)\times 3$ (**Must be evaluated**)

Question Number	Scheme	Marks
2	$g(x)=\dfrac{2x^2-5x+8}{x-2}$	
(a)	Sets $2x^2-5x+8=\left(Ax+B\right)(x-2)+C$ with an attempt at one constant	M1
	Attempts all 3 constants. E.g. Sets $x=2\Rightarrow C=...,-2B+C=8\Rightarrow B=...$ and compares x^2 terms gives $A=...$	dM1
	$2x-1+\dfrac{6}{x-2}$	A1
		(3)
(b)	$\int 2x-1+\dfrac{6}{x-2}\,dx=x^2-x+6\ln\left(x-2\right)$	M1 A1ft
	$\int_4^8 2x-1+\dfrac{6}{x-2}\,dx=\left[x^2-x+6\ln\left(x-2\right)\right]_4^8=56+6\ln 6-12-6\ln 2$	
	$=44+6\ln 3$	dM1 A1
		(4)
		(7 marks)

ANSWERS IN (a) of $A=2,B=-5,C=2$, and/or $28+2\ln 3$ in (b) please send to review

(a) **Note that correct values for *A*, *B*, and *C* imply the first two marks**

M1: Attempts to find one of the constants via

- an identity e.g. $2x^2-5x+8=\left(Ax+B\right)(x-2)+C$

  Note that if an identity is used it must be correct. Condone arithmetical slips when evaluating.
  The identity may be implied by correct equations
  e.g. $x^2: A=2,\ x: -2A+B=-5,\ \text{constant}: -2B+C=8$

- division is used look for a quotient as far as $2x$

$$\begin{array}{r} 2x-1 \\ x-2\overline{\big)\,2x^2-5x+8} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ 6 \end{array}$$

Note that the division may appear in many different formats or may be by inspection so $2x$ is sufficient to score this mark.

e.g. $\dfrac{2x^2-5x+8}{x-2}=\dfrac{2x(x-2)+4x-5x+8}{x-2}=2x+\dfrac{-(x-2)-2+8}{x-2}=2x-1+\dfrac{6}{x-2}$

e.g.

	$2x$	-1	
x	$2x^2$	$-x$	6
-2	$-4x$	2	

dM1: Attempts to find all 3 constants which are all non-zero. It is dependent upon the previous mark. Via an identity: If three simultaneous equations are formed allow this mark to be scored for proceeding to values for *A*, *B* and *C*. You do not need to check this. Allow sign slips only in forming the equations. Via division: Look for a quotient of $2x \pm c$ and a constant remainder

A1: $2x - 1 + \frac{6}{x-2}$ The expression must be written; it cannot be awarded for just stating the correct values of *A*, *B* and *C*, unless it is later written correctly as an expression in (b) (may be seen within an integral)

(b)

M1: Attempts to integrate the term $\frac{C}{x-2}$ proceeding to $D\ln|x-2|$ *C* does not have to be equal *D* (do not be concerned with brackets or modulus sign)

A1ft: Correct integration of $\mathrm{g}(x)$ for their *A*, *B* and *C* in part (a).

They must be integrating an expression of the form $Ax + B + \frac{C}{x-2} \Rightarrow \frac{Ax^2}{2} + Bx + C\ln|x-2|$ o.e (where $A, B, C \neq 0$) which may be unsimplified with or without a constant of integration.

Do not be concerned by the absence of the bracket or modulus on the $x - 2$ (as both values substituted in will give positive lns)

dM1: Attempts to proceed to the form $p + q\ln r$ by

- using the limits 8 and 4 in an expression of the form $Ex^2 + Fx + D\ln|x-2|$ o.e. (where $D, E, F \neq 0$) and subtracting either way round
- combining their ln terms **correctly**. e.g. this mark cannot be scored for $\ln 6 - \ln 2 = \frac{\ln 6}{\ln 2} = \ln 3$
  but treat as a sign slip in front of the log rather than incorrect log work e.g. $6\ln 6 - 6\ln 2 = -6\ln 3$

Condone slips when evaluating or multiplying out brackets only (condone a spurious + c present for this mark). The substitution and combining of their ln terms can be implied by their $p + q\ln r$ provided no incorrect log work is seen. Note that ln *r* does not have to be ln 3 e.g. it may be ln 729.

A1: $44 + 6\ln 3$ Withhold the final mark if there is an integral and d*x* around the answer or + *c*

Alt (b) Attempts via substitution

Note there may be attempts using substitution e.g. $u = x - 2$ on **some or all** of their expression.

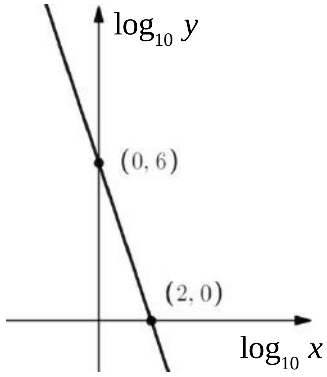
e.g. $\int_4^8 \frac{6}{x-2}\,\mathrm{d}x \to \int_2^6 \frac{6}{u}\,\mathrm{d}u = [6\ln u]_2^6$

M1: Correct attempt to integrate $\frac{C}{u}$ to $D\ln|u|$ *C* does not have to be equal *D*

A1ft: $A(u+2) + B + \frac{C}{u} \Rightarrow \frac{Au^2}{2} + (2A+B)x + C\ln|u|$ or condone $\frac{Ax^2}{2} + Bx + C\ln|u|$

dM1: Attempts to either substitute in the limits 6 and 2, subtracts either way round and combines their ln terms correctly, or substitutes back to an expression in terms of *x* and proceeds as in the main scheme and notes. If they are working in a mixture of variables, $x^2 - x + 6\ln u$ then the correct limits must be substituted into the appropriate terms.

A1: $44 + 6\ln 3$ Withhold the final mark if there is an integral and d*x* around the answer or + *c*

Question Number	Scheme	Marks
3 (i)	$y=\frac{10^6}{x^3}\Rightarrow\log_{10}y=\log_{10}10^6-\log_{10}x^3\Rightarrow\log_{10}y=6-3\log_{10}x$	
		B1 B1 B1 **(3)**
(ii)	States $\log_3 N = 2t+4$ **or** $\log_3 N=\log_3 a+t\log_3 b$	B1
	$N=3^{2t+4}=3^{2t}\times3^4$ **or** $\log_3 a=4\Rightarrow a=...$ **or** $\log_3 b=2\Rightarrow b=...$	M1
	$N=81\times9^t$	A1cso **(3)** **(6 marks)**

(i) **Condone invisible brackets around coordinates provided the meaning is clear.**

Coordinates may be stated away from the graph which is fine provided a graph is drawn.
Note that they may draw a curve so B0B1B1 is possible as the marks are earned independently of each other.
Note that if they draw on Figure 2 the axis labels are incorrect so maximum score is B0B1B1

B1: For a straight line with negative gradient anywhere on a set of axes. It must intersect both axes and not stop at these points. Do not penalise if the line is not perfectly straight and do not be concerned with the steepness provided there is no intention to draw a curve or a horizontal line. Axes do not need to be labelled but if they are then the *y*-axis needs to be $\log y$ and the horizontal axis must be $\log x$ (with or without base 10 labelled)

B1: For an intercept of 6 on the $\log_{10} y$ axis for their graph. Can be labelled as just 6 instead of $(0, 6)$. The line must pass through (or start/stop at) this point. Condone *y* intercept labelled as $(6, 0)$ provided the point is on the positive *y*-axis. Do not be concerned by the axis labelling for this mark.

B1: For an intercept of 2 on the $\log_{10} x$ axis for their graph. Can be labelled as just 2 instead of $(2, 0)$. The line must pass through (or start/stop at) this point. Condone *x* intercept being labelled as $(0, 2)$ provided the point is on the positive *x*-axis. Do not be concerned by the axis labelling for this mark.

(ii) **Note that $N = 81\times 9^t$ with no working is 0 marks**

May work in terms of *x* instead of *t* which is fine, but the final answer must be in terms of *t*

A misread writing a different base to 3 (including 10) can score a maximum B0M1A0.

B1: For a correct equation $\log_3 N = 2t+4$ which may be left unsimplified or states $\log_3 N = \log_3 a + t\log_3 b$

Condone lack of base 3 written but base 10 is B0. May be implied by further work which is not the final answer.

If they write down an incorrect equation e.g. $\log_3 N = -2t+4$ as well as $\log_3 N = \log_3 a + t\log_3 b$ then allow this mark to be scored.

M1: For using the laws of logs correctly and moving from $\log_3 N = at+b$ to $N = 3^{at}\times 3^b$

If they write $3^{2t}+3^4$ then M0A0 but if it is ambiguous between a + and × give the benefit of the doubt.

This may involve using their $\log_3 N = at+b$ for two different values for *t* to find *N* using logs correctly. They then form two simultaneous equations using $N = ab^t$ and proceed to find a value for *a* or *b*.

e.g. $t=1\Rightarrow \log_3 N = 6\Rightarrow N = 729,\ t=2\Rightarrow \log_3 N = 8\Rightarrow N = 6561$

$$\Rightarrow ab = 729,\ ab^2 = 6561\Rightarrow b = \frac{6561}{729} \text{ or } a = 81$$

e.g. $(0, 81),\ (-2, 1)\Rightarrow 81 = ab^0,\ 1 = ab^{-2}\ \Rightarrow a = 81 \text{ or } b = 9$

Alternatively, forms an equation in *a* or *b* and proceeds to find a value for *a* or *b* using logs correctly.

(may be implied by their values for *a* or *b* **provided no incorrect log work is seen**)

If they misread and write a different base this mark can be scored for working correctly in that base.

e.g. $\log_{10} a = 4\Rightarrow a = 10^4$ o.e. or e.g. $N = 10^4\times 10^{2t}$

A1: $N = 81\times 9^t$ cso **Just stating the values of *a* and *b* does not score this mark. Must be in terms of *t***
We must see

- $\log_3 N = 2t+4$ or $N = 3^{2t+4}$ or $N = 3^{2t}\times 3^4$ before proceeding to the answer with no incorrect log work seen
- $\log_3 N = \log_3 a + t\log_3 b$ **or** both $\log_3 a = 4\Rightarrow a = 3^4$ o.e. **and** $\log_3 b = 2\Rightarrow b = 3^2$ o.e. before proceeding to $N = 81\times 9^t$

Note that e.g. $N = 3^{2t+4} = 3^{2t}+3^4 \Rightarrow N = 81\times 9^t$ is B1M0A0 (incorrect log work – the values of 81 and 9 cannot imply the method mark as we see incorrect log work)

Question Number	Scheme	Marks
	$\mathrm{f}(x)=8\sin x\cos x+4\cos^2 x-3$	
4(a)	States or uses $\sin 2x=2\sin x\cos x$ or $\cos 2x=\pm 2\cos^2 x\pm 1$	M1
	Uses $\sin 2x=2\sin x\cos x$ and $\cos 2x=\pm 2\cos^2 x\pm 1$ in f (x)	dM1
	$\left(\mathrm{f}(x)=\right)8\sin x\cos x+4\cos^2 x-3=4\sin 2x+2\cos 2x-1$	A1
		(3)
(b)	$R^2=a^2+b^2\Rightarrow R=\sqrt{20}$ or $2\sqrt{5}$	B1ft
	$\tan\alpha=\dfrac{b}{a}\Rightarrow\alpha=...\ \left(=\text{"awrt } 0.464\text{"}\right)$	M1
	$\left(\mathrm{f}(x)=\right)\ 2\sqrt{5}\sin\left(2x+0.464\right)-1$	A1
		(3)
(c)	(i) Maximum value = $\text{"}2\sqrt{5}-1\text{"}$	B1 ft
	(ii) Solves $2x+\alpha=\dfrac{5\pi}{2}\Rightarrow x=...$	M1
	$(x=)$ awrt 3.69 (or $(x=)$ awrt 3.70)	A1
		(3)
		(9 marks)

(a) For the method marks condone working in mixed variables provided the intention is clear

M1: **Either** states one of the correct identities $\sin 2x=2\sin x\cos x$ or $\cos 2x=2\cos^2 x-1$

or uses $\sin 2x=2\sin x\cos x$ or $\cos 2x=\pm 2\cos^2 x\pm 1$ (may be implied by use of e.g. $\cos^2 x=\pm 1\pm\sin^2 x$ and $\cos 2x=\pm 1\pm 2\sin^2 x$ or $\cos 2x=\pm\cos^2 x\pm\sin^2 x$)

Can be implied by **either of**

- $a=4$
- $b=\pm 2$ **and** $c=-5$ or -1

dM1: **Uses** $\sin 2x=2\sin x\cos x$ and $\cos 2x=\pm 2\cos^2 x\pm 1$ (which may be implied as above for the first method mark) in f (x) to produce an expression of the required form $a\sin 2x+b\cos 2x+c$
Condone slips when substituting in.

Can be implied by $a=4$ **and** $b=\pm 2$ **and** $c=-5$ or -1

A1: $4\sin 2x+2\cos 2x-1$ Correct answer scores all three marks. Must be in terms of *x*.

(b) **Full marks can only be scored provided correct values for *a*, *b* and *c* are found in (a).**

B1ft: Finds the exact value of *R* using $R^2=a^2+b^2$ (or may use trigonometry using their value for α)

Can be implied by a correct exact value.

Follow through on their *a* or *b* but the correct values should give $R=\sqrt{20}$ or $2\sqrt{5}$

M1: Uses a "correct " method to find α using their *a* and *b*. Accept for example $\tan\alpha = \pm\frac{b}{a} \Rightarrow \alpha = ...$ or $\tan\alpha = \pm\frac{a}{b} \Rightarrow \alpha = ...$ (You may need to check this on your calculator if only the angle is seen)

May also be seen using e.g. $\sin\alpha = \pm\frac{b}{"R"} \Rightarrow \alpha = ...$ or $\cos\alpha = \pm\frac{a}{"R"} \Rightarrow \alpha = ...$ Allow the angle to be found in degrees for this mark. $\alpha = $"awrt $26.6°$" (1dp)

A1cso: $\left(\text{f}(x) =\right)\ 2\sqrt{5}\sin\left(2x + \text{awrt}\, 0.464\right) - 1$ o.e. e.g. $\left(\text{f}(x) =\right)\ \sqrt{20}\sin\left(2x + 0.464\right) - 1$ (may be awarded if seen in (c)) Can only be scored provided correct values for *a*, *b* and *c* are found in (a).

(c) (i)

B1ft: "$2\sqrt{5} - 1$" o.e. e.g. "$\sqrt{20} - 1$" but follow through on their $R + c$ as long **as *R* has been correctly found** (ie awarded in (b)). Ft on their *c* which may be different in (a) and (b) but **cannot be 0**

Allow to be a decimal if *R* was given as a decimal in (b) at any point. isw following a correct answer.

(c)(ii)

M1: Solves $2x + \alpha = \frac{5\pi}{2} \Rightarrow x = ...$ using their α found in (b) (note that this is still the equation which will need to be solved if they differentiate f(x) first and set equal to 0). You may need to check this on your calculator.

Allow this mark to be scored even if there are additional equations formed (and possibly solved).

They must be consistent in their use of degrees or radians in an equation that they are solving.

$\frac{5\pi}{2} = 7.85...$ may be used in their working.

A1: $(x =)$ awrt 3.69 but also allow $(x =)$ awrt 3.70. If several angles are found then they must indicate which angle is their final answer by e.g. underlining, circling

Question Number	Scheme	Marks
5 (a)	$f^{-1}(22) \Rightarrow 2+5\ln x = 22 \Rightarrow \ln x = 4 \Rightarrow (x=)\ e^4$	M1 A1
		(2)
(b)	$g(x) = \dfrac{6x-2}{2x+1} \Rightarrow g'(x) = \dfrac{6(2x+1)-2(6x-2)}{(2x+1)^2}$	M1 A1
	States $(g'(x)=)\ \dfrac{10}{(2x+1)^2} > 0$ hence increasing *	A1*
		(3)
(c)	$y = \dfrac{6x-2}{2x+1} \Rightarrow 2xy + y = 6x - 2 \Rightarrow 2xy - 6x = -y - 2$	M1
	$\Rightarrow x = \dfrac{-y-2}{2y-6}$ So $g^{-1}(x) = \dfrac{-x-2}{2x-6}$ o.e.	A1
	Domain $0 < x < 3$	B1
		(3)
(d)	Range fg is $fg < 2 + 5\ln 3$	M1, A1
		(2)
		(10 marks)

(a)

M1: Sets $2+5\ln x = 22$ and rearranges to $\ln x = ...$ (may be implied by further work)

Alternatively, may attempt to find $f^{-1}(x)$ and substitute 22 into the expression. i.e. $f^{-1}(x) = e^{\frac{\pm x \pm 2}{5}}$.

Maybe implied by awrt 55.

Condone sign slips only in their rearrangement.

A1: e^4 (must be simplified and exact) isw once a correct answer is seen.

(b)

M1: Either

- attempts to use the quotient rule proceeding to the form $\dfrac{A(2x+1)-B(6x-2)}{(2x+1)^2}$ o.e. where A and B are both positive. Do not award this mark if the quotient rule is applied the wrong way round.
- attempts to use the product rule proceeding to the form $(6x-2)(2x+1)^{-1} \Rightarrow \pm C(6x-2)(2x+1)^{-2} \pm D(2x+1)^{-1}$
- writes $g(x)$ as $3 - \dfrac{5}{2x+1} = 3 - 5(2x+1)^{-1}$ and attempts to use the chain rule proceeding to the form $E(2x+1)^{-2}$

In all methods invisible brackets can be recovered or implied by further work.

A1: Correct unsimplified e.g. $\dfrac{6(2x+1)-2(6x-2)}{(2x+1)^2}$ or $-2(6x-2)(2x+1)^{-2} + 6(2x+1)^{-1}$ or $10(2x+1)^{-2}$

A1*: Requires

- a reason as to why the expression is always positive e.g. by finding $\left(\mathrm{g}'(x)=\right)\dfrac{10}{(2x+1)^2}>0$
- a minimal conclusion

e.g. writes $\mathrm{g}'(x)$ as $\dfrac{10}{(2x+1)^2}$ **and deduces > 0** with minimal conclusion e.g. "it is an increasing function" or "hence increasing" **Condone use of** $\geqslant$

They may consider just the denominator separately, but they would still need to simplify the numerator to 10 and state e.g. $(2x+1)^2>0$ o.e. with a minimal conclusion

Do not accept the conclusion $\mathrm{g}'(x)$ is an increasing function if stated instead of $\mathrm{g}(x)$.

Note that they cannot just state e.g. "$-2(6x-2)(2x+1)^{-2}+6(2x+1)^{-1}>0$ so increasing" as it is not clear from the expression that this will be the case.

Do not withhold this mark if invisible brackets are recovered but A0* if there are errors in the method.

(c)

M1: Attempts to change the subject. Look for cross multiplication with an attempt to collect terms achieving the form $\pm 2xy\pm 6x=\pm y\pm 2$ or $\pm 2xy\pm 6y=\pm x\pm 2$. May be implied by further work.

If they start from their $y=\alpha\pm\dfrac{\beta}{2x+1}$ look for an alternative rearrangement to $\dfrac{\beta}{y\pm\alpha}=2x+1$ (or $\dfrac{\beta}{x\pm\alpha}=2y+1$)

A1: $\mathrm{g}^{-1}(x)=\dfrac{-x-2}{2x-6}$ or equivalent such as $\mathrm{g}^{-1}(x)=\dfrac{x+2}{6-2x}$ or may be seen as $\mathrm{g}^{-1}(x)=\dfrac{\frac{5}{3-x}-1}{2}$ or $\mathrm{g}^{-1}(x)=\dfrac{5}{6-2x}-0.5$

Accept $\mathrm{g}^{-1}:x\to\dfrac{x+2}{6-2x}$ Condone $\mathrm{g}^{-1}=\ldots$ or $\mathrm{g}^{-1}=y=\ldots$ but do not allow just $y=\dfrac{x+2}{6-2x}$

B1: Correct domain $0<x<3$ o.e. e.g. $0<x\cap x<3$ or $]0,3[$ but not $0<y<3$ or $0<\mathrm{g}^{-1}(x)<3$

Note: It is also acceptable to define g^{-1} in any variable e.g. as $\mathrm{g}^{-1}(t)=\dfrac{t+2}{6-2t}$, $0<t<3$ as long as the variable is used consistently to score M1A1B1. If another variable is used other than *x* it must be fully defined e.g. $\mathrm{g}^{-1}(t)=\ldots$ not just $\mathrm{g}^{-1}=\ldots$

(d)

M1: $2+5\ln 3$ found. Allow awrt 7.5 instead of $2+5\ln 3$ for this mark only. Do not be concerned by the direction of the inequality if seen.

A1: Fully correct. $(-\infty<)\ \mathrm{fg}<2+5\ln 3$ o.e. $5\ln 3$ must be exact. Do not be too concerned by alternative notation to fg as long as it is not *x*. There are many alternatives including $y<2+5\ln 3$ or $(-\infty,\ 2+5\ln 3)$ but not $[-\infty,\ 2+5\ln 3]$ as requires a strict inequality at the upper bound. Do not allow $\mathrm{f}<2+5\ln 3$ or $\mathrm{g}<2+5\ln 3$

Question Number	Scheme	Marks
6 (a)	$y=(4x-7)^{\frac{1}{2}} \Rightarrow \left(\frac{dy}{dx}=\right) 2(4x-7)^{-\frac{1}{2}}$ (see notes)	M1 A1
	At $(8,5)$ gradient of tangent is $2(4\times 8-7)^{-\frac{1}{2}}\left(=\frac{2}{5}\right)$	dM1
	Equation for *l* is $y-5=-\frac{5}{2}(x-8)$	ddM1
	$2y-10=-5x+40 \Rightarrow 5x+2y-50=0$ *	A1*
		(5)
(b)	$\int (4x-7)^{\frac{1}{2}}\,dx=\left[\frac{(4x-7)^{\frac{3}{2}}}{6}\right]$ (see notes)	M1, A1
	Complete area = $\int_{\frac{7}{4}}^{8}(4x-7)^{\frac{1}{2}}\,dx=\left[\frac{(4x-7)^{\frac{3}{2}}}{6}\right]_{\frac{7}{4}}^{8}+5$	dM1
	$=\frac{155}{6}$	A1
		(4)
		(9 marks)

(a)

M1: Attempts to differentiate to achieve $a(4x\pm 7)^{-\frac{1}{2}}$ or equivalent. The index does not need to be processed.

Alternatively, attempts to differentiate implicitly $y^2=4x-7 \Rightarrow Ay\frac{dy}{dx}=B$

A1: Achieves $2(4x-7)^{-\frac{1}{2}}$ o.e. (which may be unsimplified but the index processed) or using implicit differentiation achieves $2y\frac{dy}{dx}=4$

dM1: Substitutes x = 8 into their $a(4x\pm 7)^{-\frac{1}{2}}$ or substitutes y = 5 into their $Ay\frac{dy}{dx}=B$. It does not need to be evaluated. It is dependent on the first method mark but allow this mark to be scored if they have incorrectly manipulated their derivative before substituting in, or made transcription errors including if they lose the $-\frac{1}{2}$ index from the bracket.

ddM1: A full method for the equation of *l*. Look for

- substitution of x = 8 (or $y=5$ if implicit differentiation) into an attempt at a derivative
- the application of the negative reciprocal rule
- the use of $(8,5)$ with a correct gradient for their value of *m* to form an equation for the normal.

$y-5=\text{"}-\frac{5}{2}\text{"}(x-8)$ with the coordinates in the correct positions. If $y=-\frac{1}{m}x+c$ is used they must proceed as far as $c=...$

It is dependent on both of the previous two method marks.

A1*: Correctly achieves $5x+2y-50=0$ (in any order on the same side of the equation).
There must be a correct intermediate stage of working following their initial equation for *l* before achieving the given answer, e.g. $y-5=-\frac{5}{2}\left(x-8\right)\Rightarrow 2y-10=-5x+40\Rightarrow 5x+2y-50=0$

(b) **Note that if no integration is seen then 0 marks.**

M1: Integrates to achieve a correct form $b\left(4x\pm7\right)^{\frac{3}{2}}$ o.e. (the index does not need to be processed)

May use substitution e.g. $u=4x\pm7\Rightarrow\int\frac{u^{\frac{1}{2}}}{4}\,\mathrm{d}u\Rightarrow Qu^{\frac{3}{2}}$ (the integral must be correct but allow a slip on the coefficient for the integrated expression)

A1: Correct integration $\frac{\left(4x-7\right)^{\frac{3}{2}}}{6}$ (which may be unsimplified) with or without a constant of integration.
Condone poor notation e.g. if the integral sign and/or d*x* is still present. Index needs to be processed.

Using the substitution method score for e.g. $\frac{u^{\frac{3}{2}}}{6}$ where $u=4x-7$

dM1: Full method to find the shaded area. Note the area of the triangle is $\frac{1}{2}\times(10-8)\times5=5$

Look for $\left[b\left(4x\pm7\right)^{\frac{3}{2}}\right]_{\frac{7}{4}}^{8}+5$ o.e. (Provided this is seen then it is sufficient to proceed to the answer)

The limits must be correct for the integration
May see integration used for the triangle
e.g. $\int_{\frac{7}{4}}^{8}\left(4x\pm7\right)^{\frac{1}{2}}\,\mathrm{d}x+\int_{8}^{10}\left(-\frac{5}{2}x+25\right)\mathrm{d}x\Rightarrow\left[b\left(4x\pm7\right)^{\frac{3}{2}}\right]_{\frac{7}{4}}^{8}+\left[-\frac{5}{4}x^{2}+25x\right]_{8}^{10}$
It is dependent on the previous method mark, but you do not need to see the limits substituted in.
If they do not use the given equation of the straight line *l* then this mark cannot be scored.
Condone poor notation provided the intention is clear.

If using the substitution you may see e.g. $\left[\frac{u^{\frac{3}{2}}}{6}\right]_{0}^{25}+5$ (send to review if unsure)

A1: $\frac{155}{6}$ or exact equivalent e.g. $25\frac{5}{6}$ or $25.8\dot{3}$ provided all the previous marks have been scored.

Question Number	Scheme	Marks
7 (a)	$\sqrt{2}\sin\left(x+45°\right)=\cos\left(x-60°\right)$	
	$\sqrt{2}\left(\sin x\cos 45°+\cos x\sin 45°\right)=\cos x\cos 60°+\sin x\sin 60°$	M1 A1
	$\sin x+\cos x=\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x$	
	$\cos x=\left(\sqrt{3}-2\right)\sin x$	M1
	$\tan x\ \left(=\frac{1}{\sqrt{3}-2}=\frac{\sqrt{3}+2}{-1}\right)=-2-\sqrt{3}$ *	A1*
		(4)
(b)	States or uses $x+45°=2\theta$ o.e.	B1
	Proceeds from e.g. $\tan\left(2\theta-45°\right)=-2-\sqrt{3}\Rightarrow 2\theta-45°=105°,285°$	M1
	Correct order of operations to find one angle	dM1
	$\theta=75°,165°$	A1
		(4)
		(8 marks)

(a) Condone working in mixed variables

M1: Attempts to use the compound angle identities to produce an equation in $\sin x$ and $\cos x$

Look for $\sqrt{2}\times\left(\sin x\cos 45°\pm\cos x\sin 45°\right)=\cos x\cos 60°\pm\sin x\sin 60°$ o.e.

May be implied by a first line of e.g. $\sin x\pm\cos x=\frac{1}{2}\cos x\pm\frac{\sqrt{3}}{2}\sin x$ o.e.

Condone missing brackets.

A1: Correct equation in $\sin x$ and $\cos x$.

E.g. $\sqrt{2}\left(\sin x\cos 45°+\cos x\sin 45°\right)=\cos x\cos 60°+\sin x\sin 60°$ o.e. (may be implied by further work). May also see use of equivalent angles e.g. $\cos\left(-60\right)°=\cos 60°$

M1: For an attempt to solve the problem. Look for an attempt (condoning slips) at two of the three elements required to complete the proof, namely

- use of correct exact trigonometric values for $\sin 45°,\cos 45°,\cos 60°,\sin 60°$
- collection of like terms in $\sin x$ and $\cos x$ or $\tan x$
- use of $\tan x=\frac{\sin x}{\cos x}$

A1*: Proceeds to the given answer with all previous marks scored.

There should be no errors in the manipulation and no bracket omissions, other than a missing trailing bracket.

e.g. $\sqrt{2}(\sin x\cos 45°+\cos x\sin 45°=\cos x\cos 60°+\sin x\sin 60°$

When proceeding from $A\sin x=B\cos x$ (where A or B may be 1) to the given answer it is acceptable to proceed from $\frac{1}{2}\cos x=\frac{-2+\sqrt{3}}{2}\sin x$ to the given answer as evidence of the use of $\tan x=\frac{\sin x}{\cos x}$.

Condone poor notation to be recovered provided the final answer line is written correctly. Condone working in mixed variables provided the final given answer is all in terms of *x*.

Allow surd work to be done via a calculator.

(b)

B1: States or uses $x+45°=2\theta$ o.e. e.g. $\theta=\frac{x+45°}{2}$

This is implied for sight of the equation $\tan\left(2\theta-45°\right)=-2-\sqrt{3}$

M1: Proceeds from $\tan\left(2\theta\pm\alpha°\right)=-2-\sqrt{3}\Rightarrow 2\theta\pm\alpha°=105°$ or $285°$ where $\alpha\neq 0$

The attempt must either achieve an angle of $105°$ or $285°$ or equivalent expression (Allow in radians (3sf) which are 1.83, 4.97), or allow a general solution of e.g. $2\theta\pm\alpha°=-75°+\left(180n\right)°$

Maybe implied by further work which is not $75°$ or $165°$

dM1: Correct order of operations to solve their $2\theta\pm\alpha°=...$

This is dependent on the previous method mark.

Note that $\tan\left(2\theta-45°\right)=-2-\sqrt{3}\Rightarrow 75°$ does not imply this mark. We must see either e.g. $2\theta-45°=105°\Rightarrow\theta=...$ or some intermediate stage before seeing $75°$

A1: Both angles $\theta=75°,165°$ with no others given within the range

Note that $\tan\left(2\theta-45°\right)=-2-\sqrt{3}\Rightarrow 2\theta-45°=105°\Rightarrow\theta=75°,165°$ is acceptable for full marks

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Alt (b) via use of $\cos(2\theta-105°)=\cos 2\theta\cos 105°+\sin 2\theta\sin 105°$

$$\sqrt{2}\sin 2\theta=\cos(2\theta-105°)\Rightarrow\tan 2\theta=\frac{\cos 105°}{\sqrt{2}-\sin 105°}\Rightarrow\theta=75°,165°$$

Note the order of the marks needs to match up to the main scheme so 0110 is possible.

B1: For achieving $\tan 2\theta=-\frac{\sqrt{3}}{3}$ o.e. so allow $\tan 2\theta=\frac{\cos 105°}{\sqrt{2}-\sin 105°}=$ awrt -0.58

Or via double angle identities $\sqrt{3}\tan^2\theta-6\tan\theta-\sqrt{3}=0$ o.e.

M1: Attempts to use the compound angle identities (allowing sign slips when using them) to reach a form $\tan 2\theta=k$ where k is a constant not $-2-\sqrt{3}$ (or expression in trigonometric terms such as $\cos 105°$ as seen above). Allow $2\theta=-30°$ o.e. (allow in radians $-\frac{\pi}{6}$) to imply this mark. Do not be concerned by the mechanics of their rearrangement.

Alternatively, via double angle identities reaches a 3TQ in $\tan\theta$

dM1: Correct order of operations from $\tan 2\theta=k$ proceeding to $\theta=...$ e.g. $2\theta=-30°\Rightarrow\theta=-15°$ **(which must be in degrees)** can score this mark. You may need to check this if $\theta=\frac{\tan^{-1}k}{2}=...$ is not written

Alternatively, correctly solves their $\sqrt{3}\tan^2\theta-6\tan\theta-\sqrt{3}=0$ proceeding to $\theta=...$

A1: Both angles $\theta=75°,165°$ with no others given within the range

Question Number	Scheme	Marks
8	$h=1.5x-0.5x\mathrm{e}^{0.02x}$	
(a)	$0=1.5d-0.5d\,\mathrm{e}^{0.02d}\Rightarrow \mathrm{e}^{0.02d}=3$	M1
	$\Rightarrow 0.02d=\ln 3\Rightarrow d=54.93$	dM1 A1
		(3)
(b)	$\left(\dfrac{\mathrm{d}h}{\mathrm{d}x}=\right)1.5-\left(0.5\mathrm{e}^{0.02x}+0.5x\times 0.02\mathrm{e}^{0.02x}\right)$	M1A1
	Sets $0=1.5-0.5\mathrm{e}^{0.02x}-0.5x\times 0.02\mathrm{e}^{0.02x}\Rightarrow \mathrm{e}^{0.02x}\left(0.5+0.5x\times 0.02\right)=1.5$	
	$\Rightarrow \mathrm{e}^{0.02x}=\dfrac{1.5}{\left(0.5+0.5x\times 0.02\right)}$	dM1
	$\Rightarrow \mathrm{e}^{0.02x}=\dfrac{1.5}{\left(0.5+0.5x\times 0.02\right)}=\dfrac{150}{50+x}\Rightarrow x=50\ln\left(\dfrac{150}{50+x}\right)$ *	A1*
		(4)
(c)	(i) awrt 31.43	M1 A1
	(ii) awrt 30.88 metres **(including units)**	A1
		(3)
		(10 marks)

(a) **Allow to be solved in either *x* or *d***

M1: Sets $h=0$, cancels by d (or takes out a factor of d) and proceeds to a form $a\mathrm{e}^{\pm 0.02d}=b$ where $ab>0$ and a may be 1. Condone a misread or miscopy proceeding to the form $a\mathrm{e}^{\pm 0.2d}=b$

dM1: Solves the equation of the required form using logs. It is dependent on the previous method mark. Look for an expression for d (or x) or a multiple of d (or x) before proceeding to a value for d.

e.g. $\mathrm{e}^{0.02d}=3\Rightarrow d=\dfrac{\ln 3}{0.02}$

e.g. $\mathrm{e}^{-0.02d}=\dfrac{1}{3}\Rightarrow -0.02d\ln \mathrm{e}=\ln\dfrac{1}{3}\Rightarrow d=$

Condone the slip on the index to be $\pm 0.2d$

It cannot be implied by a numerical value for *d*.

A1: 54.93 (m) cao (units not required but if given they must be correct) following correct log work seen and at least one intermediate stage of working. Condone $x=54.93$. isw if they round after a correct answer is seen.

Minimum acceptable e.g. $\mathrm{e}^{0.02d}=3\Rightarrow d=\dfrac{\ln 3}{0.02}=54.93$ (M1dM1A1)

e.g. $0=1.5d-0.5d\,\mathrm{e}^{0.02d}\Rightarrow 0.02d=\ln 3\Rightarrow d=54.93$ (M1dM1A1)

e.g. $0=1.5d-0.5d\,\mathrm{e}^{0.02d}\Rightarrow 3=\mathrm{e}^{0.02d}\Rightarrow d=54.93$ (M1dM0A0)

(b) **If no attempt is seen for (b) then allow differentiation seen in (a) to score in (b).**

M1: Attempts the product rule on $x\mathrm{e}^{0.02x}$ achieving $\pm Ax\mathrm{e}^{0.02x} \pm B\mathrm{e}^{0.02x}$ where A can be 1. It is likely to be part of an expression.

Condone a misread or miscopy proceeding to the form $\pm Ax\mathrm{e}^{0.2x} \pm B\mathrm{e}^{0.2x}$

A1: $\left(\frac{\mathrm{d}h}{\mathrm{d}x}=\right)1.5-\left(0.5\mathrm{e}^{0.02x}+0.5x\times0.02\mathrm{e}^{0.02x}\right)$ o.e.

dM1: Sets $0=1.5-"0.5\mathrm{e}^{0.02x}-0.5x\times0.02\mathrm{e}^{0.02x}"$, attempts to make $\mathrm{e}^{\pm0.02x}$ (or $C\mathrm{e}^{\pm0.02x}$) the subject and proceeds to the form $C\mathrm{e}^{\pm0.02x}=\frac{D}{E+Fx}$ where C can be 1.

May see $0=150-"50\mathrm{e}^{0.02x}-x\mathrm{e}^{0.02x}" \Rightarrow \mathrm{e}^{0.02x}=\frac{150}{50+x}$

Condone slips in their rearrangement but they must take out a factor of $\mathrm{e}^{\pm0.02x}$ (or $C\mathrm{e}^{\pm0.02x}$) and divide by their bracket. If they take logs of both sides first, the rearrangement must be correct.

Condone invisible brackets to be recovered or implied by further work.

Condone a misread or miscopy proceeding from $0=1.5-"0.5\mathrm{e}^{0.02x}-0.5x\times0.02\mathrm{e}^{0.02x}"$ and attempting to make $\mathrm{e}^{\pm0.2x}$ (or $C\mathrm{e}^{\pm0.2x}$) the subject.

A1*: Achieves the given answer with no errors seen provided all previous marks have been scored.

Do not allow this mark to be scored for proceeding directly from

$\mathrm{e}^{0.02x}\left(0.5+0.5x\times0.02\right)=1.5$ to $x=50\ln\left(\frac{150}{50+x}\right)$.

We must see either $\mathrm{e}^{0.02x}=\frac{1.5}{\left(0.5+0.5x\times0.02\right)}$ o.e. or an unsimplified expression for x

e.g. $x=50\ln\left(\frac{1.5}{0.5+0.01x}\right)$ before achieving the given answer.

(c)(i) **Check by the question. If there is a contradiction between answers, the answer in the main body of the work takes precedence.**

M1: Attempts the iteration formula "correctly" seen once.

E.g. Award for $x_2=50\ln\left(\frac{150}{80}\right)$ or awrt 31.4

A1: awrt 31.43

(c)(ii)

A1: awrt 30.88 m **(including units)**

Question Number	Scheme	Marks
9 (a)	$(k=)\ 4\sin^2\left(\frac{\pi}{3}\right)-1=2$ *	B1*
		(1)
(b)	(i) $x=4\sin^2 y-1 \Rightarrow \frac{dx}{dy}=8\sin y\cos y$ o.e.	M1 A1
	(ii) Attempts either $\sin^2 y=\frac{\pm x\pm1}{4}$ or $\cos^2 y=\pm1\pm\frac{x+1}{4}$ (both for dM1)	M1 dM1
	$\frac{dy}{dx}=\frac{1}{8\sin y\cos y}=\frac{1}{8\times\sqrt{\frac{x+1}{4}}\times\sqrt{\pm1\pm\frac{x+1}{4}}}=\frac{1}{2\sqrt{x+1}\sqrt{3-x}}$ *	ddM1 A1*
		(6)
(c)	At $x=2$, gradient of curve = $\frac{1}{2\sqrt{3}} \Rightarrow$ gradient of normal is $-2\sqrt{3}$	M1
	Point N (base length of triangle) is solution of $\cancel{y}-\frac{\pi}{3}=-2\sqrt{3}(x-2) \Rightarrow x=2+\frac{\pi}{6\sqrt{3}}$ (= 2.3...)	dM1
	Area = $\frac{1}{2}\times\left(2+\frac{\pi}{6\sqrt{3}}\right)\times\frac{\pi}{3}=\frac{\pi}{3}+\frac{\pi^2}{36\sqrt{3}}$	A1
		(3)
		(10 marks)

(a)

B1*: Verifies that $k=2$ with no errors seen. (Condone $x=2$) Do not allow this mark if rounded numbers are seen in their solution for e.g. $\frac{\pi}{3}$. They may work in degrees and substitute in $60°$ which is acceptable. Look for as a minimum e.g. $(k=)\ 4\sin^2\frac{\pi}{3}-1=2$ Do not allow $k=4\sin\left(\frac{\pi}{3}\right)^2-1=2$

If they rearrange the equation to e.g. $y=\sin^{-1}\left(\sqrt{\frac{x+1}{4}}\right)$ and substitute in $x=2$ proceeding to e.g. $\frac{\pi}{3}$ then they must conclude that $k=2$ (or a preamble followed by a minimal conclusion e.g. QED, tick)

(b) (i)

M1: Differentiates to a form $p\sin y\cos y$ or equivalent. Indices do not need to be processed for this mark. e.g. $...\sin^{2-1} y\cos y$

May use the identity $\cos 2y=\pm1\pm2\sin^2 y \Rightarrow x=4\sin^2 y-1=1-2\cos 2y \Rightarrow \frac{dx}{dy}=q\sin 2y$

A1: Correct lhs and rhs. For $\frac{dx}{dy}=8\sin y\cos y$ or $4\sin 2y$. Note that $\frac{dy}{dx}=...$ is A0. isw for this mark after a correct answer is seen. You may see $\frac{dx}{dy}$ (or allow x') on an earlier line which is fine for this mark. May be seen at the start of (b)(ii). Indices must be processed for this mark.

(b)(ii) **On EPEN this is M1A1dM1A1* we are marking this as M1dM1ddM1A1***
Note this is a given answer so full working must be seen.

M1: Attempts to find either $\sin y$ (or $\sin^2 y$) **or** $\cos y$ (or $\cos^2 y$) in terms of *x* (or possibly multiples of these) Typically look for $\sin^2 y = \frac{\pm x \pm 1}{4}$ or $\sin y = \sqrt{\frac{\pm x \pm 1}{4}}$ Also allow e.g. $4\sin^2 y = \pm x \pm 1$

May attempt to use $\pm\sin^2 y \pm \cos^2 y = \pm 1 \Rightarrow \cos^2 y = \pm 1 \pm \sin^2 y \Rightarrow \cos^2 y = \pm 1 \pm \frac{x \pm 1}{4}$ or

$\cos y = \sqrt{\pm 1 \pm \frac{x \pm 1}{4}}$ Ignore $\pm\sqrt{...}$ for this mark. (may be seen within their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$)

dM1: Attempts to find both $\sin y$ (or $\sin^2 y$) **and** $\cos y$ (or $\cos^2 y$) or possibly multiples of these in terms of *x*. (See previous method mark for guidance) It is dependent on the previous method mark.

ddM1: Full method to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ of the form $\frac{1}{p\sin y\cos y}$ o.e. in terms of *x* e.g. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{"8"\sqrt{\frac{\pm x \pm 1}{4}}\sqrt{\pm 1 \pm \frac{\pm x \pm 1}{4}}}$

Condone square root notation which does not fully go over any fraction provided it is implied by further work which is not the given answer. Condone missing arguments for sin and cos provided the intention is clear or implied. Condone missing brackets to be recovered **before** achieving the given answer. If they have rearranged or simplified incorrectly for $\cos y$ before substituting, ddM1 can still be scored.

A1*: Achieves the given answer **provided all previous method marks have been scored in (b) and no errors.** Look for

- a correct expression for $\cos^2 y$ or $\cos y$ in *x* which has not already been simplified to $...\sqrt{3-x}$
- a correct expression for $\sin^2 y$ or $\sin y$ in *x* which has not already been simplified to $...\sqrt{x+1}$

$\frac{\mathrm{d}y}{\mathrm{d}x}$ (or y') may appear on an earlier line which is acceptable. Note that if they work backwards by substituting $x = 4\sin^2 y$ into the given answer there must be a conclusion that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$ isw

(c)

M1: Full method for the gradient of the normal using the *x* or *y* value. Condone arithmetical slips but the gradient must be the negative reciprocal of the gradient of the tangent to the curve.

Note that they may use an earlier incorrect line in part (b) for $\frac{\mathrm{d}y}{\mathrm{d}x}$ (or even $\frac{\mathrm{d}x}{\mathrm{d}y}$).

Allow use of an earlier line provided it is an attempt at a correct method substituting in *x* or *y* and proceeding from e.g. $\frac{\mathrm{d}x}{\mathrm{d}y} = \alpha \Rightarrow$ gradient of the normal $= -\alpha$

May be implied by awrt -3.5 If no method is shown where the gradient of the tangent or normal has come from, then it must be correct.

dM1: Attempts to find the *x* coordinate of *N*. It is dependent on the previous method mark. Uses their gradient of the normal with $\left(2, \frac{\pi}{3}\right)$, sets $y = 0$ in $y - y_1 = m(x - x_1)$ and proceeds to find *x*. May also use $y = mx + c$ to find the equation for the straight line and then sets $y = 0$ and rearranges to find *x*. Do not be concerned by the mechanics of the rearrangement for this mark.

A1: $\frac{\pi}{3} + \frac{\pi^2}{36\sqrt{3}}$ or exact equivalent such as $\frac{\pi}{3} + \frac{\pi^2\sqrt{3}}{108}$ which is in the form $a\pi + b\pi^2$